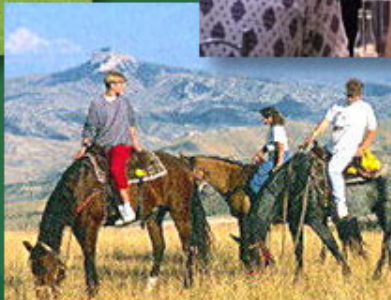


What would you do with more free time?



Introduction to Basic Reliability Statistics

James Wheeler, CMRP

The Allied logo features a stylized white 'A' with a swoosh, followed by the word 'Allied' in a bold, italicized serif font. A registered trademark symbol (®) is located at the end of the word.

Inspired Reliability

Objectives

- Arithmetic Mean
- Standard Deviation
- Correlation Coefficient
- Estimating MTBF
 - Type I Censoring
 - Type II Censoring
- Exponential Distribution
- Reliability Predictions
- Weibull Curves and Intro to Weibull Analysis
- Basic System Reliability
 - Series System
 - Active Parallel Systems



Arithmetic Mean

- The arithmetic mean or simply “mean” is the sum of a group of numbers divided by the number of items in the group.
- In statistics, this is denoted by $\bar{x}$ (pronounced “x bar”)
- Example: What is the arithmetic mean of 24,37,16 and 21?

$$\bar{x} = (24 + 37 + 16 + 21) \div 4 = 98 \div 4 = 24.5$$



Arithmetic Mean

Example

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

$$x_1 = 24$$

$$x_2 = 37$$

$$x_3 = 16$$

$$x_4 = 21$$

$$\bar{x} = \frac{1}{4} \sum_{i=1}^4 x_i$$

$$\bar{x} = \frac{1}{4} (24 + 37 + 16 + 21)$$

$$\bar{x} = \frac{1}{4} (98)$$

$$\bar{x} = \frac{98}{4}$$

$$\bar{x} = 24.5$$



Standard Deviation

- Standard deviation is the measure of statistical dispersion in a set of numbers.
- It is the Root Mean Square (RMS) of the deviation from the arithmetic mean of a group of numbers.
- If the data points are all close to the mean then the Standard Deviation is close to zero.
- If the data points are far from the mean then the standard deviation is far from zero.
- Standard deviation is noted by the lower case Greek letter Sigma (σ)



Standard Deviation

Example

$$x_1 = 24$$

$$x_2 = 37$$

$$x_3 = 16$$

$$x_4 = 21$$

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}$$

For known population size

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

Estimate for unknown population size

$$\sigma = \sqrt{\frac{1}{4} [(24 - 24.5)^2 + (37 - 24.5)^2 + (16 - 24.5)^2 + (21 - 24.5)^2]}$$

$$\sigma = 7.76208$$

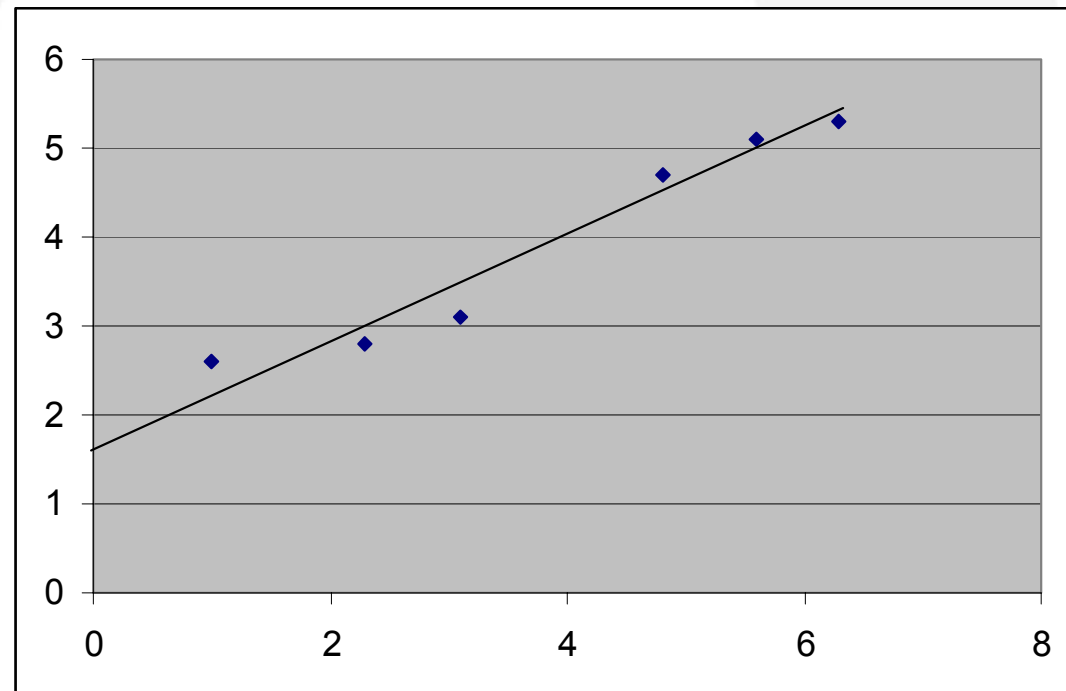


Correlation Coefficient

- Is the likelihood that 2 sets of numbers are related
 - The closer the correlation value gets to 1.0, the more linear the relationship between the 2 sets of numbers.
- It is based on calculations of slope (m), y-intercept (b) and correlation (r)

x	y
1	2.6
2.3	2.8
3.1	3.1
4.8	4.7
5.6	5.1
6.3	5.3

Slope	0.584
Y-Intercept	1.684
Correlation	0.974



Correlation Coefficient

- Using the X & Y values, there is a non-graphical method for calculating slope (m), y intercept (b) and correlation (r).

$$m = \frac{n \Sigma (xy) - \Sigma x \Sigma y}{n \Sigma (x^2) - (\Sigma x)^2}$$

$$b = \frac{\Sigma y - m \Sigma x}{n}$$

$$r = \frac{n \Sigma (xy) - \Sigma x \Sigma y}{\sqrt{[n \Sigma (x^2) - (\Sigma x)^2][n \Sigma (y^2) - (\Sigma y)^2]}}$$

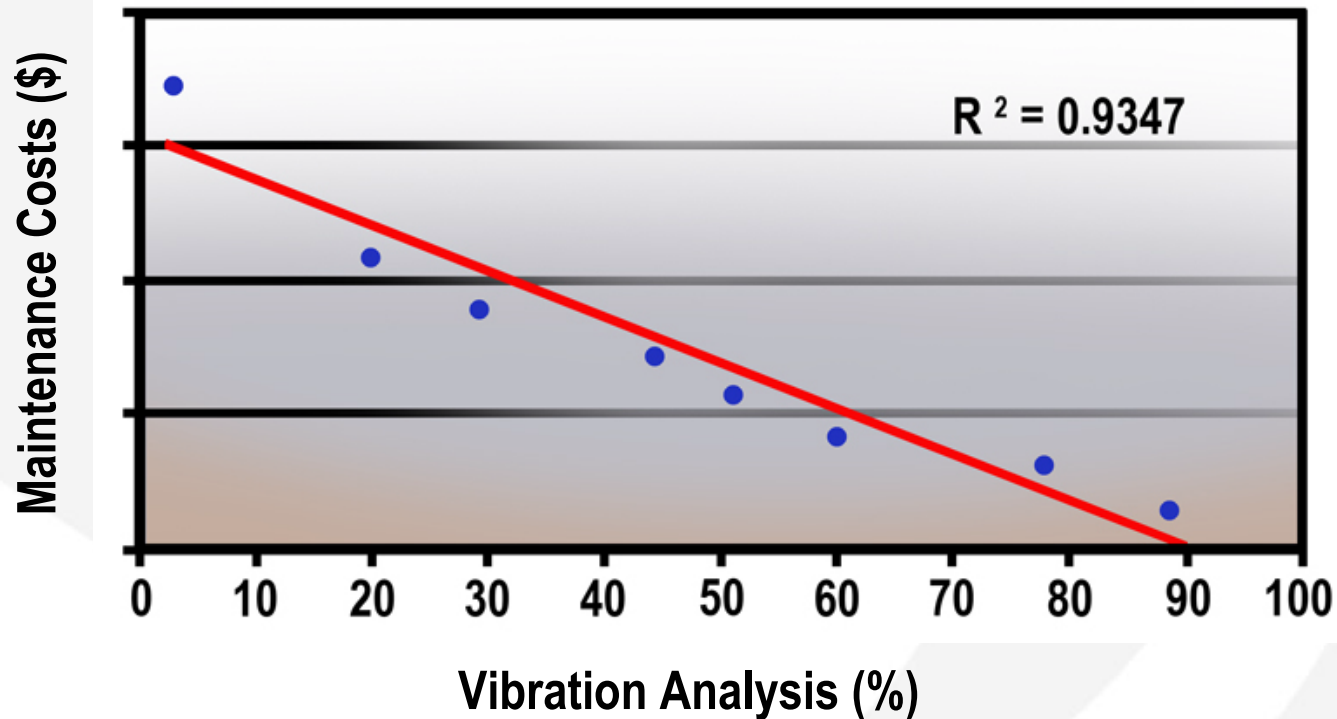


Correlation Coefficient

- Why do I need this? How can I use it?
- Does equipment become more prone to failure or more expensive failures as it ages?
 - Collect some ages and failure rate data and find out?
 - Collect some ages and MTBF and find out?
- Other examples:
 - For a pump, are motor amps and gallons per minute perfectly linear?



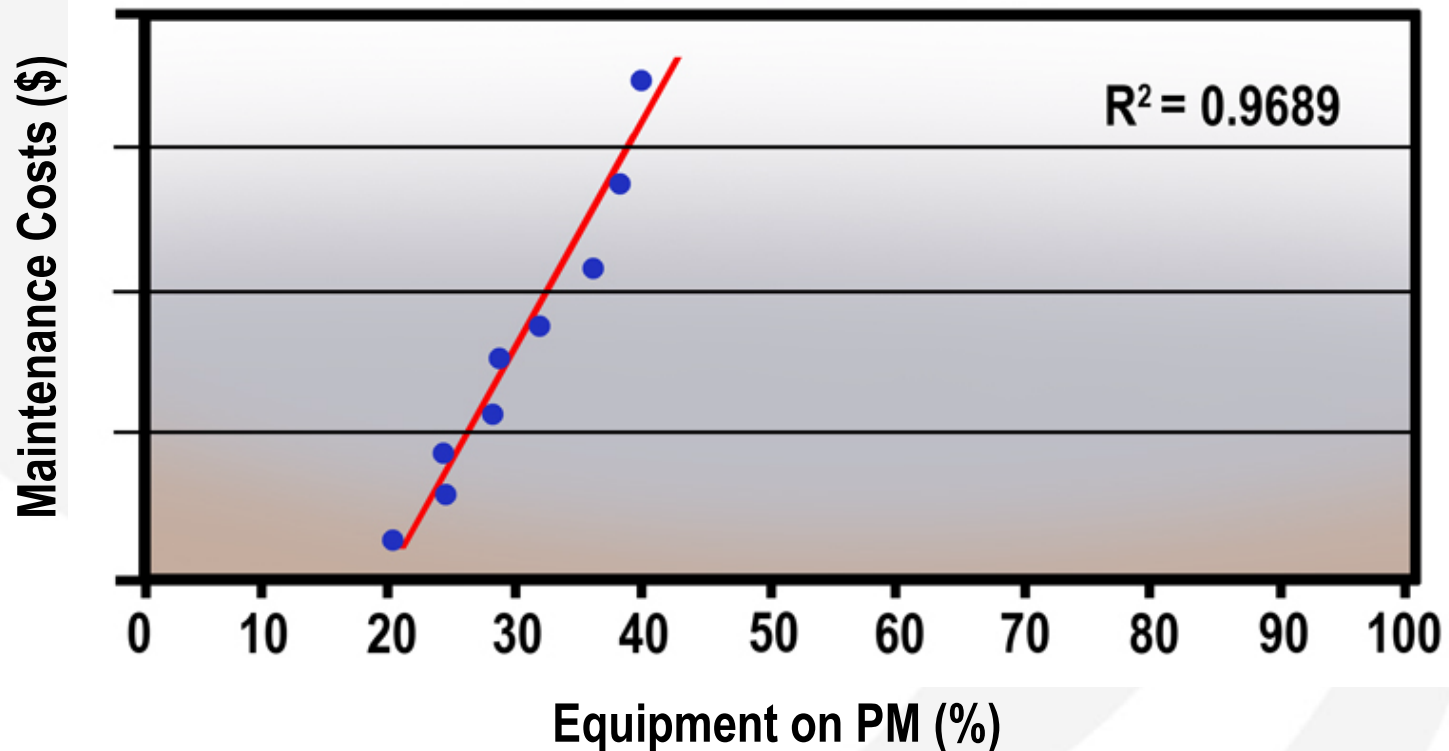
Maintenance Costs versus Vibration Analysis (PdM)



Source: 1997 Benchmarking Study in Chemical Processing industry, John Schultz to be featured in Ron Moore's new book *What Tool? When? Selecting the Right Manufacturing Improvement Strategies and Tools*



Maintenance Costs versus Equipment on PM



Source: 1997 Benchmarking Study in Chemical Processing industry, John Schultz to be featured in Ron Moore's new book *What Tool? When? Selecting the Right Manufacturing Improvement Strategies and Tools*



Mean Time Between Failures

- MTBF is supposed to be calculated for each individual asset
- Do you calculate it at your plant?



Estimating MTBF

Type I Censoring

- a.k.a. Time/Cycle Truncated Censoring
- Test is halted at a given number of hours.
- Failures during the test are immediately repaired and the test continues

$$\hat{\Theta} = \frac{nt}{r}$$

Where: $\hat{\Theta}$ = estimate of MTBF
n = number of items on test
t = total test time per unit
r = # of failures occurring during the test



Estimating MTBF

Type II Censoring

- a.k.a. Failure Truncated Censoring
- Test is halted at a given number of failures
- Failures during the test are immediately repaired and the test continues

$$\hat{\Theta} = \frac{\sum_{i=1}^r y_i + (n - r)y_r}{r}$$

$\hat{\Theta}$ = estimate of MTBF

y_i = time to failure i_{th} item

y_r = time to failure of the unit at which time is truncated

n = Total number of assets in test

r = Total number of failures



When would I use MTBF?

- Good question!
- MTBF can be used to help determine maintenance intervals.
- There is a significant flaw with this.
- What does the M in MTBF stand for?
- What does this implicitly tell you?



Reliability Predictions

- If I know a little bit about the MTBF for a particular asset...
- I can make some predictions about the life of that asset.

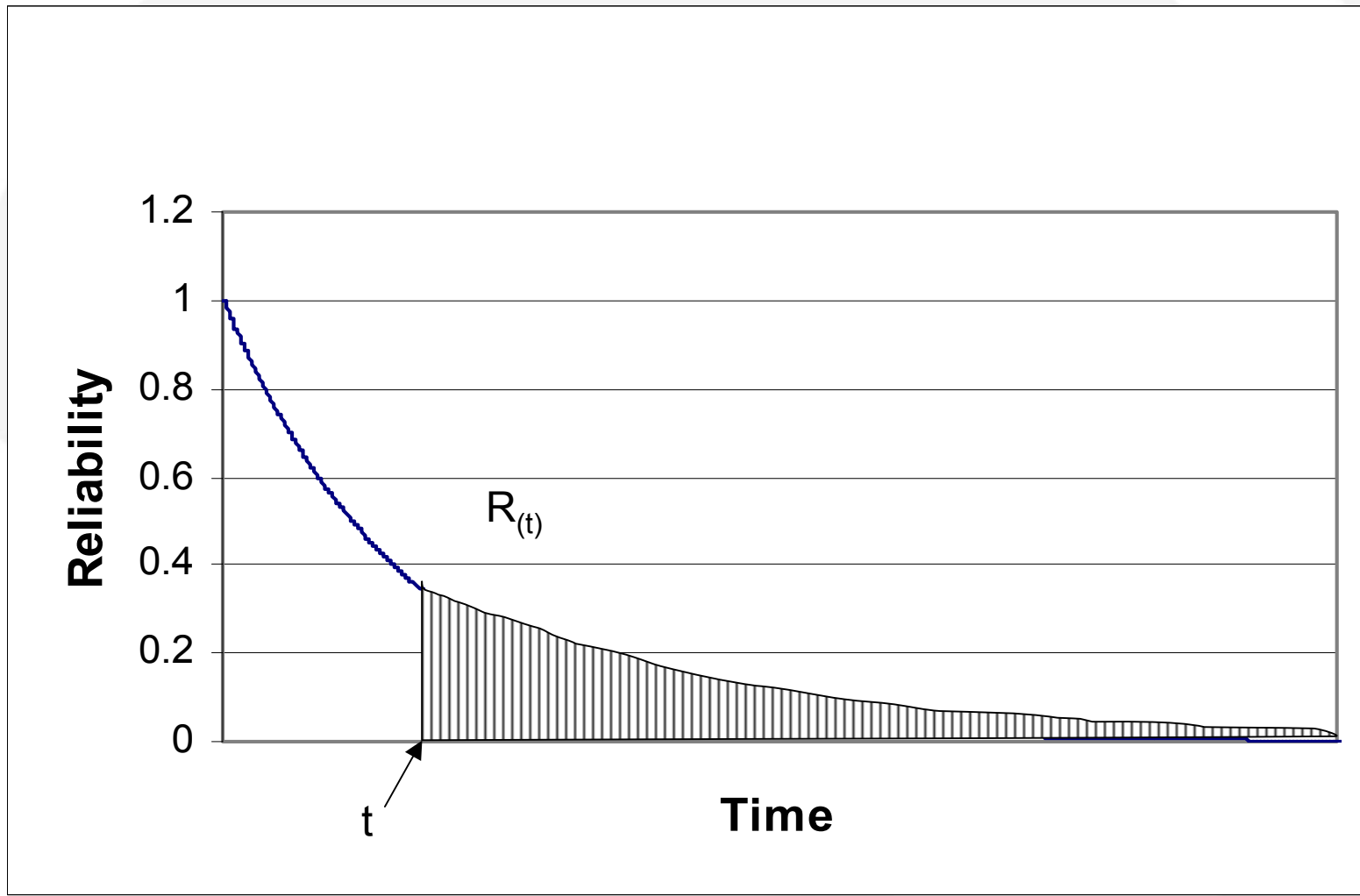


Reliability Predictions

- Q is the probability of failure.
- $Q = 1 - R$
- So then R is the probability of not failing



Exponential Distribution



Reliability Predictions

The reliability for a given time (t) during the random failure period can be calculated with the formula:

$$R_{(t)} = e^{-\lambda t}$$

Where:

e = base of the natural logarithms which is 2.718281828...

λ = failure rate (1/MTBF)

t = time



e - the base of natural logarithms

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$$

$$1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \dots$$

1	1.0000000000000000
+ 1/1!	1.0000000000000000
+ 1/2!	0.5000000000000000
+ 1/3!	0.1666666666666667
+ 1/4!	0.0416666666666667
+ 1/5!	0.0083333333333333
+ 1/6!	0.0013888888888889
+ 1/7!	0.00019841269841
+ 1/8!	0.00002480158730
+ 1/9!	0.00000275573192
+ 1/10!	0.00000027557319
e	2.71828180114638



Reliability Predictions

Example

A particular pump has a MTBF of 4,000 hours. What is the probability of operating for a period of 1,500 hours without a failure?

$$\lambda = 0.00025 \text{ or } 1/4,000$$

$$t = 1,500$$

$$e^{-\lambda t} = e^{-(0.00025)(1,500)} = e^{-0.375} = 0.68728$$

68.73% Probability exists of operating 1,500 hours without a failure exists when the MTBF = 4,000 hours.

31.27% Probability exists of a failure before operating 1,500 hours.



Reliability Predictions

If the reliability for a given time (t) during the random failure period can be calculated with the formula:

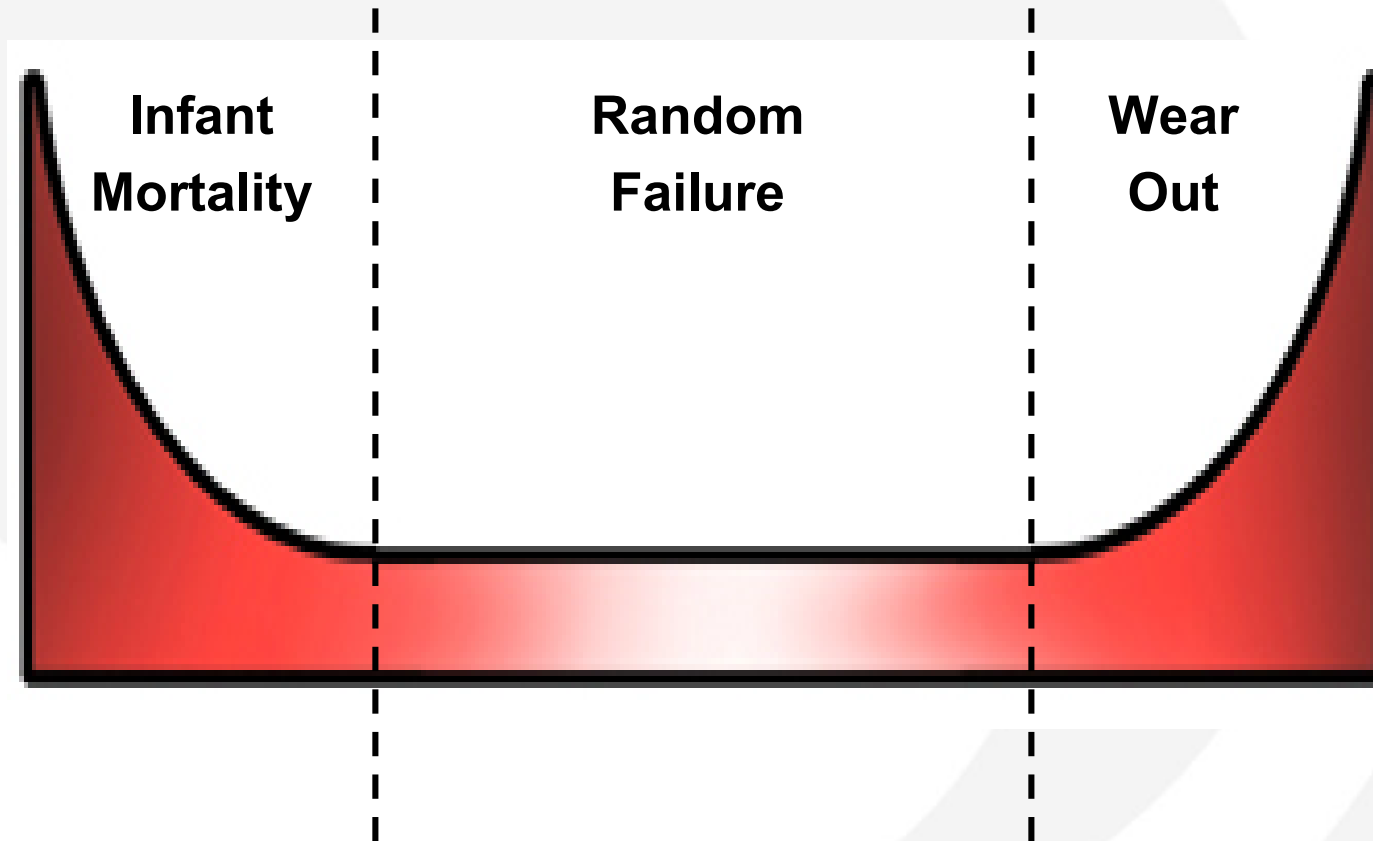
$$R_{(t)} = e^{-\lambda t}$$

Then what is the equation when I am not in the random failure period?
What if I in the infant mortality period or wear-out period?

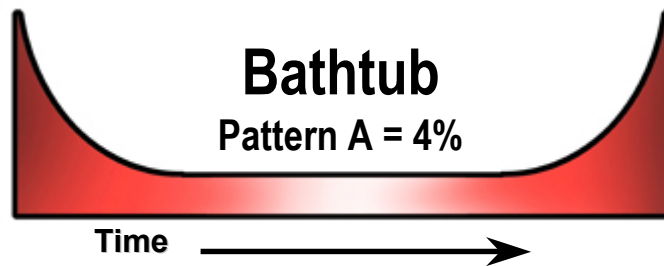
Then the equation is slightly more difficult...



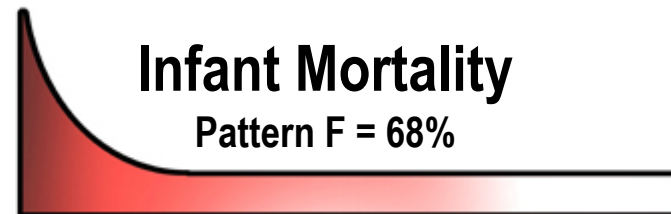
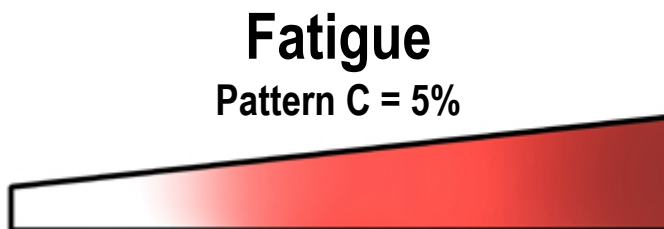
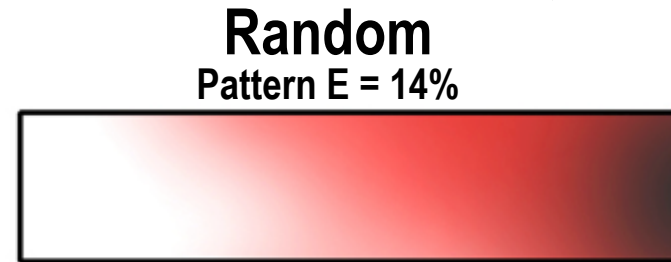
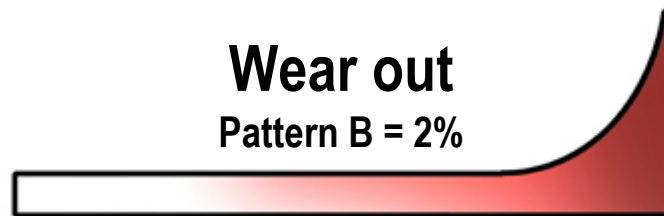
Overall (Bathtub) Curve



Weibull Shapes Individual Curves



Initial Break-in period
Pattern D = 7%

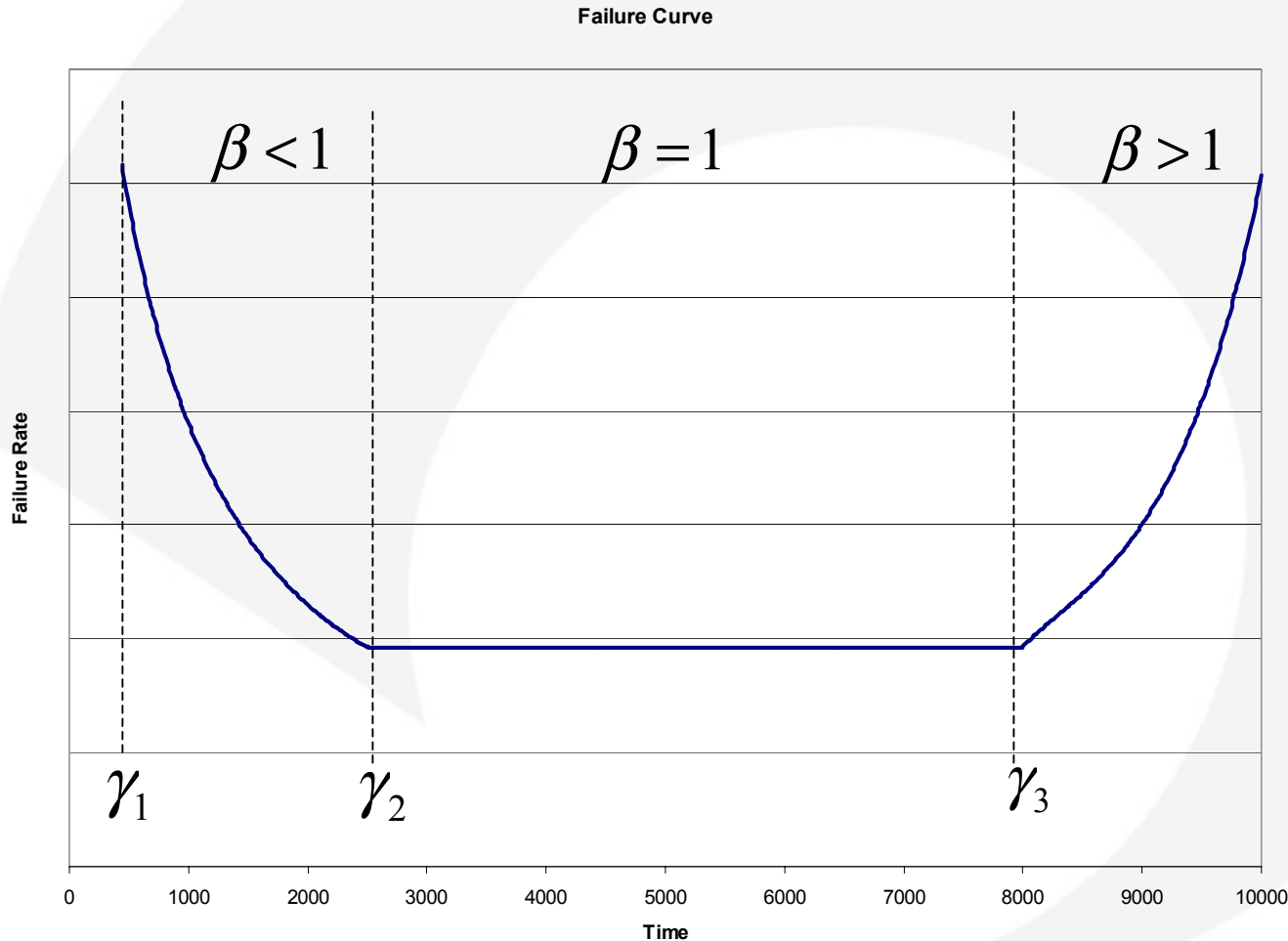


Age Related = 11%

Random = 89%



Weibull Analysis



$$R_{(t)} = e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$



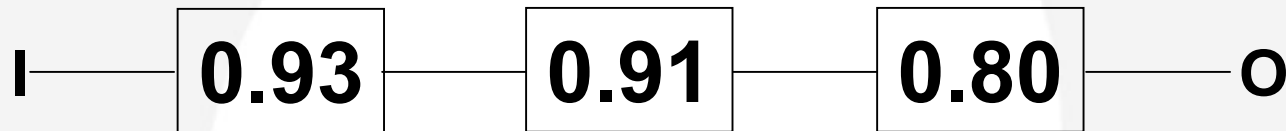
System Reliability

- Rarely do assets work alone
- Typically they are a part of a system
- Systems can many different configurations
 - Series
 - Active Parallel
 - “Hot” Standby Parallel
 - “Warm” Standby Parallel
 - “Cold” Standby Parallel
- Reliability calculations for each of these is slightly different



Series Systems - Reliability

- A system whereby the failure of a single machine shuts down the entire system is said to be a “series designed system”

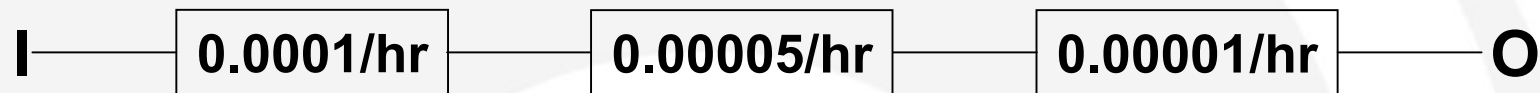


$$R_s = R_1 \times R_2 \times R_3$$

$$R_s = 0.93 \times 0.91 \times 0.80 = 0.677 \text{ or } 67.7\%$$



Series Systems – Failure Probability



$$R_{s(t)} = e^{-\sum \lambda_i t_i}$$

$$R_{s(1500)} = e^{-\sum (0.0001 + 0.00005 + 0.00001)(1500)}$$

$$R_{s(1500)} = e^{-(0.00016)(1500)}$$

$$R_{s(1500)} = e^{-(0.24)}$$

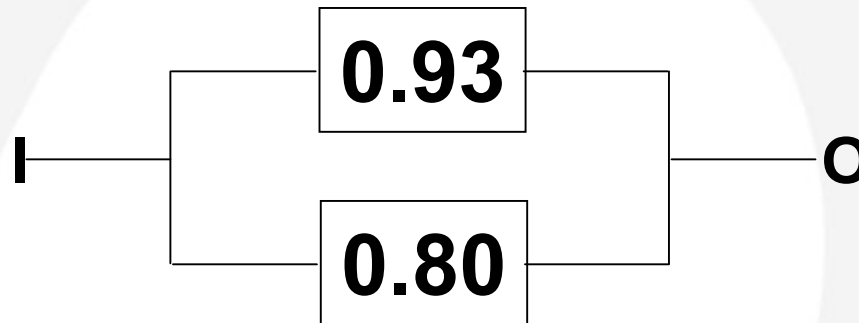
$$R_{s(1500)} = 0.7866$$

$$R_{s(1500)} = 78.6\%$$



Active Parallel Systems - Reliability

- A system where either machine can carry the full system load and a single failure does not disrupt the system is said to be an “active parallel system”

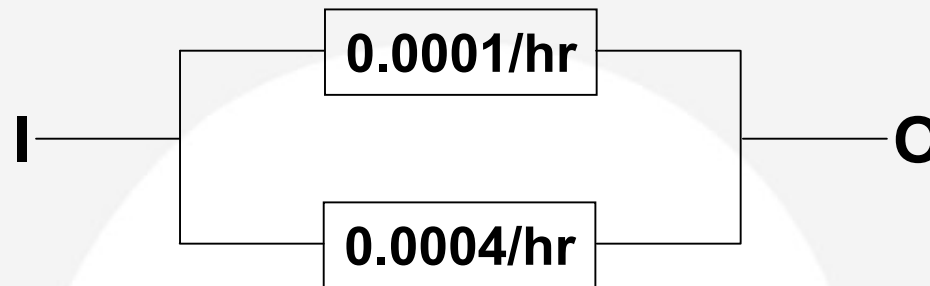


$$R_s = R_1 + R_2 - R_1 R_2$$

$$R_s = 0.93 + 0.80 - 0.93 \times 0.80 = 0.986 \text{ or } 98.6\%$$



Active Parallel Systems – Failure Probability



$$R_{(t)} = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}$$

$$R_{(1500)} = e^{-(0.0001)(1500)} + e^{-(0.0004)(1500)} - e^{-(0.0001+0.0004)(1500)}$$

$$R_{(1500)} = 0.9372$$

$$R_{(1500)} = 93.72\%$$



Questions?

Thanks!

James Wheeler, CMRP
Allied Reliability
wheelerj@alliedreliability.com

